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465. Proposed by CYRUS B. HALDEMAN, Ross, Ohio.

Having given $\tan^{-1} 1 = \tan^{-1} 1/2 + \tan^{-1} 1/3$, show that

$$\tan^{-1} 1 = 5 \tan^{-1} 1/8 + 2 \tan^{-1} 1/18 + 3 \tan^{-1} 1/57.$$

SOLUTION BY HORACE OLSON, Chicago, Illinois.

Put $\tan^{-1} 1/2 = \tan^{-1} 1/3 + \tan^{-1} x_1 = \tan^{-1}\dfrac{3x_1 + 1}{3 - x_1}$. From this equation x_1 is found to be 1/7. Hence, $\tan^{-1} 1 = 2 \tan^{-1} 1/3 + \tan^{-1} 1/7$.

Now put

$$\tan^{-1} 1/3 = \tan^{-1} 1/7 + \tan^{-1} x_2 = \tan^{-1}\frac{7x_2 + 1}{7 - x_2};$$

whence $x_2 = 2/11$, and $\tan^{-1} 1 = 3 \tan^{-1} 1/7 + 2 \tan^{-1} 2/11$.

Now put

$$\tan^{-1} 1/7 = \tan^{-1} x_3 + \tan^{-1} y_3 = \tan^{-1}\frac{x_3 + y_3}{1 - x_3y_3}.$$

This gives the indeterminate equation $x_3y_3 + 7(x_3 + y_3) = 1$, or $(x_3 + 7)(y_3 + 7) = 50$. One set of solutions of this equation is $x_3 = 1/8$, $y_3 = 1/57$. Hence, $\tan^{-1} 1 = 3 \tan^{-1} 1/8 + 3 \tan^{-1} 1/57 + 2 \tan^{-1} 2/11$. Put

$$\tan^{-1} 2/11 = \tan^{-1} 1/8 + \tan^{-1} x_4 = \tan^{-1}\frac{8x_4 + 1}{8 - x_4};$$

whence $x_4 = 1/18$. Hence, $\tan^{-1} 1 = 5 \tan^{-1} 1/8 + 2 \tan^{-1} 1/18 + 3 \tan^{-1} 1/57$.

Also solved by E. B. ESCOTT, W. J. THOME, M. T. REED, G. W. HARTWELL, and E. E. WHITEFORD.

466. Proposed by E. B. ESCOTT, Kansas City, Mo.

For what functions, f, are the following relations true:

When

$$\frac{f(x, y, z)}{X} = \frac{f(y, z, x)}{Y} = \frac{f(z, x, y)}{Z},$$

then

$$\frac{f(X, Y, Z)}{x} = \frac{f(Y, Z, X)}{y} = \frac{f(Z, X, Y)}{z}?$$

SOLUTION BY ALBERT A. BENNETT, University of Texas.

We notice that $f(X, Y, Z)$ must be homogeneous, since if we replace X, Y, Z by cX, cY, cZ, in the first relation it is unaltered, and hence also the second. Thus x, y, z may be regarded as the homogeneous coördinates of the points in one plane, X, Y, Z of those in a second. The problem may therefore be expressed as follows: What are the involutoric plane transformations with triangular symmetry? Here "involutoric" is used in the restricted sense as of period two, and triangular symmetry is used in approximately the sense first suggested by Clifford, the point (1, 1, 1) being a center of triangular symmetry. There can be little doubt that there exist transformations of this form which are essentially transcendental and which may be regarded as limiting cases of algebraic birational transformations. To catalogue the explicit forms of even the algebraic cases is perhaps out of the question, since no explicit classification of algebraic forms of Cremona transformations has been attempted beyond the very simplest cases. Normal forms under the Cremona group are indeed known. Compare Pascal's *Repertorium* or other detailed articles on birational geometry.

It is furthermore obvious that a given geometric solution gives rise to an infinite number of analytic solutions. For example, if $g(x, y, z)$ be one solution, and $\varphi(x, y, z)$ be a constant or the equation of any triangularly symmetric self-corresponding curve whether the self-correspondence be singular or not, then $\varphi(x, y, z)g(x, y, z)$ is a solution. Apart from the trivial solutions $f(x, y, z) = 0$ and $f(x, y, z) = x$, the simplest case is probably the quadratic transformation given by $f(x, y, z) = 1/x$, and then taking $\varphi(x, y, z) = xyz$, or $\sqrt{xyz}$, or $xyz/\log(1 + xyz)$ or $(xy + yz + zx)(x + y + z)$, etc., we get other related solutions from $f(x, y, z) = 1/x$.